Polynomials: DSE Revision Worksheet

Name: _____ Class: _____ No.: _____

Every question below is a *twin* of a real DSE question: same shape, new numbers. **In class:** A1–A4 (5 minutes), then B2 under exam conditions (7 minutes). **Homework:** everything else. Write Section B on foolscap, as in the DSE answer book. Target times: about **75 seconds** per MC and **1 minute per mark** for Section B.

Section A Paper 2 style (multiple choice)

A1 Let $\mathrm{p}(x) = 3x^2 + 5x + c$, where c is a constant. If $\mathrm{p}(x)$ is divisible by $x + 3$, find the remainder when $\mathrm{p}(x)$ is divided by $3x - 1$.

A. $-\dfrac{40}{3}$ B. -10 C. 10 D. 12

A2 Let $\mathrm{f}(x) = x^3 + kx^2 - 2x + 8$, where k is a constant. If $\mathrm{f}(x)$ is divisible by $x + k$, find the remainder when $\mathrm{f}(x)$ is divided by $x + 1$.

A. -5 B. 3 C. 5 D. 11

A3 Let $\mathrm{f}(x) = ax^5 + 4x^2 + bx - 7$, where a and b are constants. If $\mathrm{f}(x)$ is divisible by $x - 2$, find the remainder when $\mathrm{f}(x)$ is divided by $x + 2$.

A. -18 B. 0 C. 9 D. 18

A4 Let $\mathrm{p}(x)$ be a polynomial. When $\mathrm{p}(x)$ is divided by $x - 2$, the remainder is 9. If $\mathrm{p}(x)$ is divisible by $x + 1$, find the remainder when $\mathrm{p}(x)$ is divided by $x^2 - x - 2$.

A. $3x - 3$ B. $3x + 3$ C. $-3x + 3$ D. $-3x - 3$

A5 Let $\mathrm{f}(x) = ax^3 + 3ax^2 - 20$, where a is a constant. If $x + 2$ is a factor of $\mathrm{f}(x)$, then $\mathrm{f}(2) =$

A. -80. B. 0. C. 20. D. 80.

A6 Let k be a constant such that $3x^3 + kx^2 - 2x - 6$ is divisible by $3x + k$. Find k.

A. -9 B. 3 C. 9 D. 27

A7 Let $\mathrm{h}(x) = ax^4 + 8x^3 + b$, where a and b are constants. If $\mathrm{h}(x)$ is divisible by $2x + 1$, find the remainder when $\mathrm{h}(x)$ is divided by $2x - 1$.

A. 2 B. 1 C. 0 D. -2

A8 Let $\mathrm{g}(x) = 2x^4 + ax + b$, where a and b are constants. If $\mathrm{g}(x)$ is divisible by $x + 1$, find the remainder when $\mathrm{g}(x)$ is divided by $x - 1$.

A. 0 B. $2a$ C. $-2a$ D. $2a + 4$

Section B Paper 1 style (long questions)

Show your working. Every "Do you agree?" or "How many?" part ends with a one-line conclusion: that line carries a mark.

B1 Let $\mathrm{f}(x) = 3x^3 + hx^2 + kx - 10$, where h and k are constants. When $\mathrm{f}(x)$ is divided by $x + 1$, the remainder is -60. It is given that $\mathrm{f}(x)$ is divisible by $3x - 2$.

(a) Find h and k. (3 marks)

(b) Someone claims that all the roots of the equation $\mathrm{f}(x) = 0$ are integers. Do you agree? Explain your answer. (3 marks)

B2 When the polynomial $\mathrm{p}(x)$ is divided by $x^2 + ax - 6$, the quotient and the remainder are $2x + 1$ and $k(x + 3)$ respectively, where a and k are constants. It is given that $\mathrm{p}(x)$ is divisible by $x + 3$.

(a) Find a. (3 marks)

(b) When $\mathrm{p}(x)$ is divided by $x - 1$, the remainder is -16. How many irrational roots does the equation $\mathrm{p}(x) = 0$ have? Explain your answer. (5 marks)

B3 Let $\mathrm{p}(x) = 2x^3 + ax^2 + bx + 12$, where a and b are constants. When $\mathrm{p}(x)$ is divided by $x^2 + 3x - 1$, the remainder is $6x + 13$.

(a) Find a and b. (3 marks)

(b) Is $x + 3$ a factor of $\mathrm{p}(x)$? Explain your answer. (2 marks)

(c) Someone claims that the equation $\mathrm{p}(x) = 0$ has two irrational roots. Do you agree? Explain your answer. (3 marks)

B4 Define $\mathrm{g}(x) = x^3 + 4x^2 - 3x - 1$. Let $\mathrm{h}(x) = 2x^4 + ax^3 - 18x^2 + bx + c$, where a, b and c are constants. When $\mathrm{h}(x)$ is divided by $\mathrm{g}(x)$, the quotient and the remainder are equal.

(a) Find the quotient when $\mathrm{h}(x)$ is divided by $\mathrm{g}(x)$. (3 marks)

(b) How many rational roots does the equation $\mathrm{h}(x) = 0$ have? Explain your answer. (4 marks)

B5 When the polynomial $\mathrm{g}(x)$ is divided by $x^2 - x + a$, the quotient and the remainder are $x + 9$ and $b(x + 2)$ respectively, where a and b are constants. It is given that $\mathrm{g}(x)$ is divisible by $x + 2$.

(a) Find a. (3 marks)

(b) Is there only one possible value of b such that at least two roots of the equation $\mathrm{g}(x) = 0$ are equal? Explain your answer. (5 marks)

B6 Let $\mathrm{f}(x) = 6x^3 - 5x^2 - 21x + 7$. When $\mathrm{f}(x)$ is divided by $3x^2 + ax - 2$, the quotient and the remainder are $2x - 5$ and $bx + c$ respectively, where a, b and c are constants.

(a) Find a. (3 marks)

(b) Let $\mathrm{g}(x)$ be a quadratic polynomial such that when $\mathrm{g}(x)$ is divided by $3x^2 + ax - 2$, the remainder is $bx + c$.

i. Prove that $\mathrm{f}(x) - \mathrm{g}(x)$ is divisible by $3x^2 + ax - 2$.

ii. Someone claims that all the roots of the equation $\mathrm{f}(x) - \mathrm{g}(x) = 0$ are negative. Do you agree? Explain your answer.

(5 marks)

Self-check after marking: ☐ I wrote the conclusion sentence every time ☐ I checked the linear factor's root, too ☐ I used Δ or the quadratic formula, not "it doesn't factorise"