Polynomials: what the DSE actually asks

Polynomials has come up in every DSE since 2012: one MC in Paper 2 and one 6–8 mark question in Paper 1 Section A(2), about 8 marks in total. For a Level 5 candidate these are marks you must not drop.

1 Recent papers at a glance

Year	P1 Q	Marks	P1 part (a): way in	P1 last part asks	P2 MC shape
2017	14	8	Compare coefficients	roots of $f - g = 0$ all integers?	A
2018	12	7	Remainder + factor theorems	at least one irrational root?	C
2019	11	6	Unknown quotient $ax + b$	how many rational roots?	B
2020	13	8	Division algorithm + substitute a root	all roots integers?	A
2021	12	7	Division algorithm + substitute a root	prove a factor; all roots real?	D
2022	14	8	Compare coefficients	is it a factor?; two irrational roots?	B
2023	13	7	Compare coefficients (quotient = remainder)	how many rational roots?	C
2024	14	8	Constant term, then remainder + factor theorems	how many irrational roots?	B
2025	10	6	Remainder + factor theorems	all roots rational?	A
2026	14	8	Division algorithm + substitute a root	equal roots: only one value of b?	C

The pattern: part (a) finds the unknown constants (3 marks). In every paper above, the last part asks about the nature of the roots: integer, rational, irrational, real, or equal.

2 The three tools

Division algorithm

$f(x) = d(x)\,Q(x) + R(x)$, where $\deg R < \deg d$

d = divisor, Q = quotient, R = remainder

Remainder theorem

$f(x) \div (ax - b)$ leaves remainder $f\left(\frac{b}{a}\right)$

$2x + 1$: substitute $x = -\frac{1}{2}$

Factor theorem

$ax - b$ is a factor of $f(x) \iff f\left(\frac{b}{a}\right) = 0$

"divisible by" means "is a factor of"

The degree rule. Write unknown parts with the right degree: dividing by a quadratic leaves a remainder $mx+n$; a cubic divided by a quadratic has a linear quotient; a quadratic divided by a quadratic has a *constant* quotient k.

3 Paper 2: one MC, four shapes

Every recent MC follows the same idea: *a factor gives you an equation, then you find a remainder or a value.*

Shape	What you see	Key move
A Numbers	$p(x) = 2x^2 - 11x + c$ divisible by $x - 7$; remainder $\div (2x + 1)$? (2017)	$p(7) = 0 \Rightarrow c = -21$; then $p(-\frac{1}{2}) = -15$.
B Unknown in the divisor	$x^3 + kx^2 + 5x + 10$ divisible by $x + k$ (2024; also 2019, 2022)	Substitute $x = -k$: the k^3 terms cancel, so $k = 2$.
C Too many unknowns	$g(x) = hx^3 + 2x^2 + kx - 5$ divisible by $x - 4$; remainder $\div (x + 4)$? (2026; also 2018, 2023)	**Do not solve.** $g(4) = 0$ gives $64h + 4k = -27$. At $x = -4$, odd powers flip sign and even powers do not, so $g(-4) = -(64h + 4k) + 27 = 54$.
D Divide by a quadratic	$p(-1) = -2$, $p(1) = 0$; remainder $\div (x^2 - 1)$? (2021)	Let the remainder be $ax + b$; substitute both roots of the divisor: $x - 1$.

MC traps. The wrong options are almost always sign errors: $-15/15$, $2a/-2a$, $-108/108$. Check the sign of the substituted value ($x + 4 \Rightarrow x = -4$; $2x - 3 \Rightarrow x = \frac{3}{2}$) before you choose.

4 Paper 1 part (a): four ways in

Read what the question *gives* you, then write the first line that earns the 1M.

The question gives	Your first line	Years
Remainders and a factor, and f(x) written out with unknown coefficients	f$(-2) = -45$ and f$(\frac{5}{2}) = 0$ (simultaneous equations)	2018, 2024, 2025
"divided by $d(x)$, quotient $Q(x)$, remainder $R(x)$" *and* a factor $x - r$	$f(x) = d(x)Q(x) + R(x)$, then substitute $x = r$: one term vanishes. Here it is the remainder, e.g. $b(x-3)$ at $x = 3$ (2026, 2021). In 2020 it is $(x^2 - 1)$q(x) at $x = 1$.	2020, 2021, 2026
The divisor and remainder, with f(x) known except a few coefficients	$f(x) = (mx + n)\,d(x) + R(x)$; compare the highest and constant terms	2017, 2022, 2023
Divisible by a quadratic, with remainders for two linear divisors	$p(x) = (ax + b)\,d(x)$; substitute	2019

5 Paper 1 last part: the nature of the roots

The method, every year

1. **Factorise** into (linear) × (quadratic), using part (a) and the known factor. (1M)
2. **Settle the quadratic** with $\Delta = b^2 - 4ac$ or the quadratic formula. (1M)
3. **Check every root**, including the linear factor's, against the property asked, then write the conclusion sentence. (1A f.t.)

The number ladder

integers	$\ldots, -2, 0, 3, \ldots$
$\subset$ rationals	also $\frac{2}{3}$, $-\frac{4}{3}$
$\subset$ reals	also irrationals: $\frac{-5 \pm \sqrt{73}}{2}$
not real	roots when $\Delta < 0$

What Δ tells you (integer coefficients)

$\Delta < 0$	no real roots, so none rational or irrational
$\Delta = 0$	two equal rational roots
$\Delta > 0$, perfect square	two distinct rational roots
$\Delta > 0$, not a square	two irrational roots

Conclusion sentences that score (the last line carries the 1A f.t.)

- *Claim:* "Note that $\frac{1}{2}$ is a root of the equation, and it is not an integer. So not all the roots are integers. Thus, the claim is disagreed."
- *How many:* "Note that $\frac{-1 \pm \sqrt{97}}{12}$ are irrational, and 2 and -5 are not irrational. Thus, the equation has 2 irrational roots."
- *Is it a factor:* "p$(5) = 0$. Thus, $x - 5$ is a factor of p(x)."

6 The Level 5 twists

Twist	What happens	The move
Quotient = remainder (2023)	h $= q$ g $+ q$	Factor it out: h $= q$(g $+ 1$), then factorise g $+ 1$.
Equal roots (2026)	g$(x) = (x-3)(x^2 + 18x + 65 + b)$; when are two roots equal?	**Two cases.** (1) $\Delta = 0$ for the quadratic. (2) The linear root 3 is also a root of the quadratic. In 2026 that gives $b = 16$ *or* $b = -128$.
An unknown g(x) (2017)	g quadratic with the same remainder	Quotient is a constant: g $= k\,d(x) + R(x)$, so f $-$ g $= d(x)(Q(x) - k)$. One bad root disproves "all"; you never need k.
Using a remainder twice (2020)	You know the remainder $\div (x^2 - 1)$	It also gives f(-1) for free: substitute $x = -1$ into $(x^2 - 1)$q$(x) + (kx + 8)$.
Quartic as a product (2024)	F$(x) = (6x^2 + x + p)(qx^2 + rx - 10)$	Constant term $= p \times (-10)$. Factorise the quadratic that splits; use the formula on the other.

7 Traps that cost marks

- $\frac{1}{2}$ **is rational but not an integer** (2017). Read the property in the claim word by word.
- $\Delta < 0$ **does not mean "irrational".** It means no real roots, so there are no irrational roots either (2018, 2022).
- 0 **is a rational root** (2023). Count it.
- **Forgetting the linear factor's root.** It must be checked against the claim too, and in an "equal roots" question it can be the repeated one (2026).
- **No conclusion sentence, no 1A.** "Thus, the claim is disagreed." or "Thus, the equation has 2 rational roots."
- **"It cannot be factorised" is not a reason.** Show Δ or the quadratic formula; that line is the 1M.

8 In the exam

- **Time:** about 75 seconds for the MC; about 1 minute per mark in Paper 1 (6–8 minutes).
- **Check with the calculator:** once you have the constants, use CALC to evaluate f(r) at each root you found. Every root should give 0.
- **Follow-through:** a wrong constant in part (a) can still earn the method marks and the 1A f.t. in the last part. Always finish the question.